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SUMMARY

- If $y = f(x)$,then $\frac{dy}{dx}$ represents instantaneous rate of change of y with respect to x.
- If $y = f(g(t))$, then $\frac{dy}{dt} = f'(g(t)) \cdot g'(t)$ which is called the chain rule.
- The equation of tangent at (a,b) to the curve $y = f(x)$ is given by $y - b = \left(\frac{dy}{dx}\right)_{(a,b)}(x-a)$ or $y - b = f'(a)(x-a)$.
- Rolle's Theorem

  Let $f(x)$ be continuous in a closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a) = f(b)$, then there is at least one point $c \in (a,b)$ where $f'(c) = 0$.
- Lagrange's Mean Value Theorem

  Let $f(x)$ be continuous in a closed interval $[a,b]$ and differentiable on the open interval (a,b) (where $f(a)$ and $f(b)$ are not necessarily equal). Then there is at least one point $c \in (a,b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.
- Taylor's series

  Let $f(x)$ be a function infinitely differentiable at $x = a$. Then $f(x)$ can be expanded as a series in an interval $(x-a, x+a)$,of the form

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{\lfloor n}(x-a)^n = f(a) + \frac{f'(a)}{\lfloor 1}(x-a) + \cdots + \frac{f^{(n)}(a)}{\lfloor n}(x-a)^n + \cdots$$

- Maclaurin's series

  In the Taylor's series if $a = 0$, then the expansion takes the form

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{\lfloor n}(x)^n = f(0) + \frac{f'(0)}{\lfloor 1}(x) + \cdots + \frac{f^{(n)}(0)}{\lfloor n}(x^n) + \cdots$$

- The l'Hôpital's rule

  Suppose $f(x)$ and $g(x)$ are differentiable functions and $g'(x) \neq 0$ with

  $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$. Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

  $\lim_{x\to a} f(x) = \pm\infty = \lim_{x\to a} g(x)$. Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

- If the function $f(x)$ is differentiable in an open interval (a,b) then we say, if $\frac{d}{dx}(f(x)) > 0$, $\forall x \in (a,b)$ then $f(x)$ is strictly increasing in the interval (a,b).

if $\dfrac{d}{dx}\left(f(x)\right) < 0, \forall x \in (a,b)$ then $f(x)$ is strictly decreasing in the interval (a,b)

- A procedure for finding the absolute extrema of a continous function $f(x)$ on a closed interval $[a,b]$.

  Step 1 : Find the critical numbers of $f(x)$ in (a,b).

  Step 2 : Evaluate $f(x)$ at all critical numbers and at the endpoints a and b.

  Step 3 : The largest and the smallest of the values in Step 2 is the absolute maximum and absolute minimum of $f(x)$ respectively on the closed interval $[a,b]$.

- First Derivative Test

  Let $(c, f(c))$ be a critical point of function $f(x)$ that is continuous on an open interval I containing c. If $f(x)$ is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:(when moving across I from left to right)

  (i) If $f'(x)$ changes from negative to positive at c, then $f(x)$ has a local minimum $f(c)$.

  (ii) If $f'(x)$ changes from positive to negative at c, then $f(x)$ has a local maximum $f(c)$.

  (iii) If $f'(x)$ is positive on both sides of c, or negative on both sides of c then $f(x)$ has neither a local minimum nor a local minimum.

- Second Derivative Test

  Suppose that c is a critical point at which $f'(c) = 0$, that $f''(x)$ exists in a neighbourhood of c, and that $f'(c)$ exists. Then f has a relative maximum value at c if $f''(c) < 0$ and a relative minimum value at c if $f''(c) > 0$. If $f''(c) = 0$, the test is not informative.

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11. If $f(x)=\frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

12. If $u(x,y)=x^2+3xy+y-2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(1) -4 (2) -3 (3) -7 (4) 13

13. Linear approximation for $g(x)=\cos x$ at $x=\frac{\pi}{2}$ is

(1) $x+\frac{\pi}{2}$ (2) $-x+\frac{\pi}{2}$ (3) $x-\frac{\pi}{2}$ (4) $-x-\frac{\pi}{2}$

14. If $w(x,y,z)=x^2(y-z)+y^2(z-x)+z^2(x-y)$, then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ is

(1) $xy+yz+zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

15. If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal to

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

SUMMARY

- Let $f:(a,b)\to\mathbb{R}$ be a differentiable function and $x_0\in(a,b)$ then linear approximation L of f at x_0 is given by
  $L(x)=f(x_0)+f'(x_0)(x-x_0)\,\forall x\in(a,b)$
- Absolute error = Actual value – Approximate value

  Relative error $=\frac{\text{Absolute error}}{\text{Actual value}}$

  Percentage error $=$ Relative error $\times 100$

  (or)

  $\frac{\text{Absolute error}}{\text{Acutal value}}\times 100$
- Let $f:(a,b)\to\mathbb{R}$ be a differentiable function. For $x\in(a,b)$ and Δx the increment given to x, the differential of f is defined by $df=f'(x)\Delta x$.
- All the rules for limits (limit theorems) for functions of one variable also hold true for functions of several variables.
- Let $A=\{(x,y)\,|\,a<x<b,\ c<y<d\}\subset\mathbb{R}^2, F:A\to\mathbb{R}$ and $(x_0,y_0)\in A$.

(i) F has a partial derivative with respect to x at $(x_0, y_0) \in A$ if $\lim\limits_{h \to 0} \dfrac{F(x_0+h, y_0) - F(x_0, y_0)}{h}$ exists and it is denoted by $\left.\dfrac{\partial F}{\partial x}\right|_{(x_0, y_0)}$.

F has a partial derivative with respect to y at $(x_0, y_0) \in A$ if $\lim\limits_{k \to 0} \dfrac{F(x_0, y_0+k) - F(x_0, y_0)}{k}$ exists and limit value is defined by $\left.\dfrac{\partial F}{\partial y}\right|_{(x_0, y_0)}$.

- Clariant's Theorem: Suppose that $A = \{(x,y) \mid a < x < b,\ c < y < d\} \subset \mathbb{R}^2$, $F : A \to \mathbb{R}$. If f_{xy} and f_{yx} exist in A and are continuous in A, then $f_{xy} = f_{yx}$ in A.
- Let $A = \{(x,y) \mid a < x < b,\ c < y < d\} \subset \mathbb{R}^2$. A function $U : A \to \mathbb{R}$ is said to be harmonic in A if it satisfies $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0,\ \forall (x,y) \in A$. This equation is called Laplace's equation.
- Let $A = \{(x,y) \mid a < x < b,\ c < y < d\} \subset \mathbb{R}^2$, $F : A \to \mathbb{R}$ and $(x_0, y_0) \in A$.

  (i) The linear approximation of F at $(x_0, y_0) \in A$ is defined to be

  $$L(x,y) = F(x_0, y_0) + \left.\frac{\partial F}{\partial x}\right|_{(x_0, y_0)} (x - x_0) + \left.\frac{\partial F}{\partial y}\right|_{(x_0, y_0)} (y - y_0)$$

  (ii) The differential of F is defined to be $dF = \dfrac{\partial F}{\partial x} dx + \dfrac{\partial F}{\partial y} dy$ where $\Delta x = dx$ and $\Delta y = dy$.
- Suppose w is a function of two variables x, y where x and y are functions of a single variable 't' then $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x} \cdot \dfrac{dx}{dt} + \dfrac{\partial w}{\partial y} \cdot \dfrac{dy}{dt}$
- Suppose w is a function of two variables x and y where x and y are functions of two variables s and t then, $\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial x} \cdot \dfrac{\partial x}{\partial s} + \dfrac{\partial w}{\partial y} \cdot \dfrac{\partial y}{\partial s}$, $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y} \cdot \dfrac{\partial y}{\partial t}$
- Suppose that $A = \{(x,y) \mid a < x < b,\ c < y < d\} \subset \mathbb{R}^2$, $F : A \to \mathbb{R}^2$. If F is having continuous partial derivatives and homogeneous on A, with degree p, then $x\dfrac{\partial F}{\partial x} + y\dfrac{\partial F}{\partial y} = pF$.

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SUMMARY

(1) Definite integral as the limit of a sum

(i) $\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{r=1}^{n} f\left(a+(b-a)\frac{r}{n}\right)$

(ii) $\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{n} f\left(\frac{r}{n}\right) = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right).$

(2) Properties of definite integrals

(i) $\int_a^b f(x)\,dx = \int_a^b f(u)\,du$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(iv) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(v) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(vi) $\int_0^{2a} f(x)\,dx = \int_0^a [f(x)+f(2a-x)]\,dx.$

(vii) If $f(x)$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$

(ix) If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0.$

(x) If $f(2a-x) = f(x)$, then $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx.$

(xi) If $f(2a-x) = -f(x)$, then $\int_0^{2a} f(x)\,dx = 0.$

(xii) $\int_0^a x\,f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$ if $f(a-x) = f(x)$

(3) Bernoulli's Formula

$$\int uv\,dx = uv_{(1)} - u^{(1)}v_{(2)} + u^{(2)}v_{(3)} - u^{(3)}v_{(4)} + \cdots.$$

(4) Reduction Formulae

(i) $\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \begin{cases} \frac{(n-1)}{n}\times\frac{(n-3)}{(n-2)}\times\cdots\times\frac{1}{2}\times\frac{\pi}{2}, & \text{if } n = 2,4,6,\cdots \\ \frac{(n-1)}{n}\times\frac{(n-3)}{(n-2)}\times\cdots\times\frac{2}{3}, & \text{if } n = 3,5,7,\cdots \end{cases}$

(ii) If n is even and m is even,

$$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x\,dx = \frac{(n-1)}{(m+n)}\frac{(n-3)}{(m+n-2)}\frac{(n-5)}{(m+n-4)}\cdots\frac{1}{(m+2)}\frac{(m-1)}{m}\frac{(m-3)}{(m-2)}\frac{(m-5)}{(m-4)}\cdots\frac{1}{2}\frac{\pi}{2}$$

(iii) If n is odd and m is any positive integer (even or odd), then

$$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x\,dx = \frac{(n-1)}{(m+n)}\frac{(n-3)}{(m+n-2)}\frac{(n-5)}{(m+n-4)}\cdots\frac{2}{(m+3)}\frac{1}{(m+1)}.$$

(5) Gamma Formulae

(i) $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx = (n-1)!$ (ii) $\int_0^{\infty} e^{-ax} x^n dx = \dfrac{n!}{a^{n+1}}$

(6) Area of the region bounded by a curve and lines

(i) The area of the region bounded by a curve, above x-axis and the lines $x = a$ and $x = b$ is $A = \int_a^b y\,dx$.

(ii) The area of the region bounded by a curve, below x-axis and the lines $x = a$ and $x = b$ is $A = -\int_a^b y\,dx = \left|\int_a^b y\,dx\right|$.

(iii) Thus area of the region bounded by the curve to the right of y-axis, the lines $y = c$ and $y = d$ is $A = \int_c^d x\,dy$.

(iv) The area of the region bounded by the curve to the left of y-axis, the lines $y = c$ and $y = d$ is $A = -\int_c^d x\,dy = \left|\int_c^d x\,dy\right|$.

(v) If $f(x) \geq g(x)$, then area bounded by the curves and the lines $x = a,\ x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,dx = \int_a^b (y_U - y_L)\,dx$$

(vi) If $f(y) \geq g(y)$, then area bounded by the curves and the lines $y = c,\ y = d$ is

$$A = \int_c^d [f(y) - g(y)]\,dy = \int_c^d (x_R - x_L)\,dy$$

(7) Volume of the solid of revolution

(i) The volume of the solid of revolution about x-axis is $V = \pi\int_a^b y^2 dx$.

(ii) The volume of the solid of revolution about y-axis is $V = \pi\int_c^d x^2 dy$.

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21. Integrating factor of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is

(1) $\frac{1}{x+1}$ (2) $x+1$ (3) $\frac{1}{\sqrt{x+1}}$ (4) $\sqrt{x+1}$

22. The population P in any year t is such that the rate of increase in the population is proportional to the population. Then

(1) $P=Ce^{kt}$ (2) $P=Ce^{-kt}$ (3) $P=Ckt$ (4) $P=C$

23. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then

(1) $P=Ce^{kt}$ (2) $P=Ce^{-kt}$ (3) $P=Ckt$ (4) $Pt=C$

24. If the solution of the differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(1) 2 (2) -2 (3) 1 (4) -1

25. The slope at any point of a curve $y=f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through (-1,1). Then the equation of the curve is

(1) $y=x^3+2$ (2) $y=3x^2+4$ (3) $y=3x^3+4$ (4) $y=x^3+5$

SUMMARY

1. A differential equation is any equation which contains at least one derivative of an unknown function, either ordinary derivative or partial derivative.
2. The **order** of a differential equation is the highest derivative present in the differential equation.
3. If a differential equation is expressible in a polynomial form, then the integral power of the highest order derivative appears is called the **degree** of the differential equation
4. If a differential equation is not expressible to polynomial equation form having the highest order derivative as the leading term then that the degree of the differential equation is not defined.
5. If a differential equation contains only ordinary derivatives of one or more functions with respect to a single independent variable, it is said to be an ordinary differential equation (ODE).
6. An equation involving only partial derivatives of one or more functions of two or more independent variables is called a partial differential equation (PDE).
7. The result of eliminating one arbitrary constant yields a first order differential equation and that of eliminating two arbitrary constants leads to a second order differential equation and so on.
8. A solution of a differential equation is an expression for the dependent variable in terms of the independent variable(s) which satisfies the differential equation.
9. The solution which contains as many arbitrary constants as the order of the differential equation is called the **general solution**
10. If we give particular values to the arbitrary constants in the general solution of differential equation, the resulting solution is called a Particular Solution.

11. An equation of the form $f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0$ is called an equation with variable separable or simply a separable equation.

12. A function $f(x, y)$ is said to be a **homogeneous** function of degree n in the variables x and y if, $f(tx, ty) = t^n f(x, y)$ for some $n \in \mathbb{R}$ for all suitably restricted x, y and t. This is known as **Euler's homogeneity.**

13. If $f(x, y)$ is a homogeneous function of degree zero, then there exists a function g such that $f(x, y)$ is always expressed in the form $g\left(\frac{y}{x}\right)$.

14. An ordinary differential equation is said to be in homogeneous form, if the differential equation is written as $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

15. The differential equation $M(x, y)dx + N(x, y)dy = 0$ [in differential form] is said to be **homogeneous** if M and N are **homogeneous functions of the same degree**.

16. A **first order differential equation** of the form $\frac{dy}{dx} + Py = Q$.
where P and Q are functions of x only. Here no product of y and its derivative $\frac{dy}{dx}$ occurs and the dependent variable y and its derivative with respect to independent variable x occur only in the first degree.
The solution of the given differential equation (1) is given by $ye^{\int Pdx} = \int Qe^{\int Pdx} dx + C$.
Here $e^{\int Pdx}$ is known as the integrating factor (I.F.)

17. A first order differential equation of the form $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only. Here no product of x and its derivative $\frac{dx}{dy}$ occurs and the dependent variable x and its derivative with respect to independent variable y occur only in the first degree.In this case, the solution is given by $xe^{\int Pdy} = \int Qe^{\int Pdy} dy + C$.

18. If x denotes the amount of the quantity present at time t, then the instantaneous rate at which the quantity changes at time t is $\frac{dx}{dt}$.
This leads to a differential equation of the form $\frac{dx}{dt} = f(x, t)$.

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18. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is

(1) 0.24 b) 0.48 (3) 0.6 (4) 0.96

19. If in 6 trials, X is a binomial variable which follows the relation $9P(X=4) = P(X=2)$, then the probability of success is

(1)0.125 (2) 0.25 (3) 0.375 (4) 0.75

20. A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(1) $\frac{57}{20^3}$ (2) $\frac{57}{20^2}$ (3) $\frac{19^3}{20^3}$ (4) $\frac{57}{20}$

SUMMARY

- ***A random variable*** *X is a function defined on a sample space S into the real numbers $\mathbb{R}$ such that the inverse image of points or subset or interval of $\mathbb{R}$ is an event in S, for which probability is assigned.*
- A random variable X is defined on a sample space S into the real numbers $\mathbb{R}$ is called discrete random variable if the range of X is countable, that is, it can assume only a finite or countably infinite number of values, where every value in the set S has positive probability with total one.
- If X is a discrete random variable with discrete values $x_1, x_2, x_3, \ldots x_n, \ldots$, then the function denoted by $f(.)$ or $p(.)$ and defined by $f(x_k) = P(X = x_k)$ for $k = 1,2,3,\ldots n,\ldots$ is called the probability mass function of X
- The function $f(x)$ is a probability mass function if

  (i) $f(x_k) \geq 0$ for $k = 1,2,3,\ldots n,\ldots$ and (ii) $\sum_k f(x_k) = 1$
- The **cumulative distribution function** $F(x)$ of a discrete random variable X, taking the values $x_1, x_2, x_3, \ldots$ such that $x_1 < x_2 < x_3 < \cdots$ with probability mass function $f(x_i)$ is

  $$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i), \quad x \in \mathbb{R}$$
- Suppose X is a discrete random variable taking the values $x_1, x_2, x_3, \ldots$ such that $x_1 < x_2 < x_3, \ldots$ and $F(x_i)$ is the distribution function. Then the probability mass function $f(x_i)$ is given by $f(x_i) = F(x_i) - F(x_{i-1})$, $i = 1,2,3, \ldots$
- *Let S be a sample space and let a random variable $X : S \to R$ that takes any value in a set I of $\mathbb{R}$. Then X is called a* ***continuous random variable*** *if $P(X = x) = 0$ for every x in I*
- *A non-negative real valued function $f(x)$ is said to be a* ***probability density function*** *if, for each possible outcome x, $x \in [a,b]$ of a continuous random variable X having the property*

  $$P(a \leq x \leq b) = \int_a^b f(x)\, dx$$

- *Suppose $F(x)$ is the distribution function of a continuous random variable X. Then the probability density function $f(x)$ is given by*

$$f(x) = \frac{dF(x)}{dx} = F'(x),$$ *whenever derivative exists.*

- *Suppose X is a random variable with probability mass or density function $f(x)$* ***The expected value or mean or mathematical expectation of*** X, denoted by $E(x)$ or μ *is*

$$E(X) = \begin{cases} \sum_{x} xf(x) & \text{if } X \text{ is discrete} \\ \int_{-\infty}^{\infty} xf(x)\,dx & \text{if } X \text{ is continuous} \end{cases}$$

- *The* ***variance*** *of the random variable X* denoted by $V(X)$ or σ^2 (or σ_x^2) is

$$V(X) = E(X - E(X))^2 = E(X - \mu)^2$$

Properties of Mathematical expectation and variance

(i) $E(aX + b) = aE(X) + b$, where a and b are constants

Corollary 1: $E(aX) = aE(X)$	(when $b = 0$)
Corollary 2: $E(b) = b$	(when $a = 0$)

(ii) $Var(X) = E(X^2) - (E(X))^2$

(iii) $Var(aX + b) = a^2 Var(X)$ where a and b are constants

Corollary 3: $V(aX) = a^2 V(X)$	(when $b = 0$)
Corollary 4: $V(b) = 0$	(when $a = 0$)

- Let X be a random variable associated with a Bernoulli trial by defining it as X (success) = 1 and X (failure) = 0, such that

$$f(x) = \begin{cases} p & x = 1 \\ q = 1 - p & x = 0 \end{cases} \quad \text{where } 0 < p < 1$$

- X is called a Bernoulli random variable and $f(x)$ is called the Bernoulli distribution.

- *If* X is a Bernoulli's random variable which follows Bernoulli distribution with parameter p, the mean μ and variance σ^2 are

$$\mu = p \quad \text{and} \quad \sigma^2 = pq$$

- A discrete random variable X is called binomial random variable, if X is the number of successes in n-repeated trials such that

  (i) The n- repeated trials are independent and n is finite

  (ii) Each trial results only two possible outcomes, labelled as 'success' or 'failure'

  (iii) The probability of a success in each trial, denoted as p, remains constant
- The binomial random variable X equals the number of successes with probability p for a success and $q = 1 - p$ for a failure in n-independent trials, has ***a binomial distribution*** denoted by $X \sim B(n, p)$. The probability mass function of X is $f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, ..., n.$
- If X is a binomial random variable which follows binomial distribution with parameters p and n, the mean μ and variance σ^2 are $\mu = np$ and $\sigma^2 = np(1-p)$.

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SUMMARY

(1) A **binary operation** $*$ on a non-empty set S is a rule, which associates to each ordered pair (a, b) of elements a, b in S an unique element $a * b$ in S.

(2) **Commutative property:** Any binary operation $*$ defined on a nonempty set S is said to satisfy the commutative property, if $a * b = b * a, \ \forall a, b \in S$.

(3) **Associative property:** Any binary operation $*$ defined on a nonempty set S is said to satisfy the associative property, if $a * (b * c) = (a * b) * c, \ \forall a, b, c \in S$.

(4) **Existence of identity property:** An element $e \in S$ is said to be the **Identity Element** of S under the binary operation $*$ if for all $a \in S$ we have that $a * e = a$ and $e * a = a$.

(5) **Existence of inverse property:** If an identity element e exists and if for every $a \in S$, there exists b in S such that $a * b = e$ and $b * a = e$ then $b \in S$ said to be the **Inverse Element** of a. In such instance, we write $b = a^{-1}$.

(6) **Uniqueness of Identity:** In an algebraic structure the identity element (if exists) must be unique.

(7) **Uniqueness of Inverse:** In an algebraic structure the inverse of an element (if exists) must be unique.

(8) A **Boolean Matrix** is a real matrix whose entries are either 0 or 1.

(9) **Modular arithmetic:** Let n be a positive integer greater than 1 called the **modulus**. We say that two integers a and b are congruent modulo n if $b - a$ is divisible by n. In other words $a \equiv b \pmod{n}$ means $a - b = n \cdot k$ for some integer k and b is the **least non-negative integer reminder** when a is divided by n. $(0 \le b \le n - 1)$

(10) Mathematical logic is a study of reasoning through mathematical symbols.

(11) Let p be a simple statement. Then the **negation** of p is a statement whose truth value is opposite to that of p. It is denoted by $\neg p$, read as **not** p. The truth value of $\neg p$ is T, if p is F, otherwise it is F.

(12) Let p and q be any two simple statements. The **conjunction** of p and q is obtained by connecting p and q by the word **and**. It is denoted by $p \wedge q$, read as 'p conjunction q' or 'p hat q'. The truth value of $p \wedge q$ is T, whenever both p and q are T and it is F otherwise.

(13) The **disjunction** of any two simple statements p and q is the compound statement obtained by connecting p and q by the word 'or'. It is denoted by $p \vee q$, read as 'p disjunction q' or 'p cup q'. The truth value of $p \vee q$ is F, whenever both p and q are F and it is T otherwise.

(14) The **conditional statement** of any two statements p and q is the statement, 'If p, then q' and it is denoted by $p \rightarrow q$. The statement $p \rightarrow q$ has a truth value F when q has the truth value F and p has the truth value T; otherwise it has the truth value T.

(15) The **bi-conditional statement** of any two statements p and q is the statement 'p if and only if q' and is denoted by $p \leftrightarrow q$ The statement $p \leftrightarrow q$ has the truth value T whenever both p and q have identical truth values; otherwise has the truth value F.

(16) A statement is said to be a **tautology** if its truth value is always T irrespective of the truth values of its component statements. It is denoted by $\mathbb{T}$.

(17) A statement is said to be a **contradiction** if its truth value is always F irrespective of the truth values of its component statements. It is denoted by $\mathbb{F}$.

(18) A statement which is neither a tautology nor a contradiction is called **contingency**.

(19) Any two compound statements A and B are said to be **logically equivalent** or simply **equivalent** if the columns corresponding to A and B in the truth table have **identical truth values**. The logical equivalence of the statements A and B is denoted by $A \equiv B$ or $A \Leftrightarrow B$. Further note that if A and B are logically equivalent, then $A \leftrightarrow B$ must be a **tautology**.

(20) **Some laws of equivalence:**

Idempotent Laws: (i) $p \vee p \equiv p$ (ii) $p \wedge p \equiv p$.

Commutative Laws: (i) $p \vee q \equiv q \vee p$ (ii) $p \wedge q \equiv q \wedge p$.

Associative Laws: (i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$ (ii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.

Distributive Laws: (i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Identity Laws: (i) $p \vee \mathbb{T} \equiv \mathbb{T}$ and $p \vee \mathbb{F} \equiv p$

(ii) $p \wedge \mathbb{T} \equiv p$ and $p \wedge \mathbb{F} \equiv \mathbb{F}$

Complement Laws : (i) $p \vee \neg p \equiv \mathbb{T}$ and $p \wedge \neg p \equiv \mathbb{F}$

(ii) $\neg \mathbb{T} \equiv \mathbb{F}$ and $\neg \mathbb{F} \equiv \mathbb{T}$

Involution Law or Double Negation Law: $\neg(\neg p)\neg p$

de Morgan's Laws: (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \vee q) \equiv \neg p \vee \neg q$

Absorption Laws: (i) $p \vee (p \wedge q) \equiv p$ (ii) $p \wedge (p \vee q) \equiv p$

ICT CORNER

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