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MATHEMATICS

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20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $\text{adj}(\lambda A) = \lambda^n \text{adj}(A)$.

(iv) $A(\text{adj}A) = (\text{adj}A)A = |A|I$

(1) Only (i) (2) (ii) and (iii) (3) (iii) and (iv) (4) (i), (ii) and (iv)

21. If $\rho(A) = \rho([A\,|\,B])$, then the system $AX = B$ of linear equations is

(1) consistent and has a unique solution (2) consistent

(3) consistent and has infinitely many solution (4) inconsistent

22. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0,$ $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

(1) $\lambda = 7, \mu \ne -5$ (2) $\lambda = -7, \mu = 5$ (3) $\lambda \ne 7, \mu \ne -5$ (4) $\lambda = 7, \mu = -5$

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $\text{adj}(\text{adj } A)$ is

(1) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

SUMMARY

(1) Adjoint of a square matrix A = Transpose of the cofactor matrix of A.

(2) $A(\text{adj } A) = (\text{adj } A)A = |A| I_n$.

(3) $A^{-1} = \frac{1}{|A|} \text{adj } A$.

(4) (i) $|A^{-1}| = \frac{1}{|A|}$ (ii) $(A^T)^{-1} = (A^{-1})^T$ (iii) $(\lambda A)^{-1} = \frac{1}{\lambda} A^{-1}$, where λ is a non-zero scalar.

(5) (i) $(AB)^{-1} = B^{-1}A^{-1}$. (ii) $(A^{-1})^{-1} = A$

(6) If A is a non-singular square matrix of order n, then

(i) $\left(\text{adj } A\right)^{-1} = \text{adj}\left(A^{-1}\right) = \frac{1}{|A|}A$ (ii) $|\text{adj } A| = |A|^{n-1}$

(iii) adj $\left(\text{adj } A\right) = |A|^{n-2} A$ (iv) $\text{adj}(\lambda A) = \lambda^{n-1}\text{adj}(A)$, λ is a nonzero scalar

(v) $|\text{adj}\,(\text{adj } A)| = |A|^{(n-1)^2}$ (vi) $(\text{adj } A)^T = \text{adj}\left(A^T\right)$

(vii) $\text{adj}(AB) = (\text{adj}\,B)(\text{adj}\,A)$

(7) (i) $A^{-1} = \pm\frac{1}{\sqrt{|\text{adj } A|}}\text{adj } A$. (ii) $A = \pm\frac{1}{\sqrt{|\text{adj } A|}}\text{adj }\left(\text{adj } A\right)$.

(8) (i) A matrix A is orthogonal if $AA^T = A^T A = I$

(ii) A matrix A is orthogonal if and only if A is non-singular and $A^{-1} = A^T$

(8) Methods to solve the system of linear equations $AX = B$

(i) By matrix inversion method $X = A^{-1}B, |A| \neq 0$

(ii) By Cramer's rule $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}, \Delta \neq 0$.

(iii) By Gaussian elimination method

(9) (i) If $\rho(A) = \rho([A\,|\,B]) =$ number of unknowns, then the system has unique solution.

(ii) If $\rho(A) = \rho([A\,|\,B]) <$ number of unknowns, then the system has infinitely many solutions.

(iii) If $\rho(A) \neq \rho([A\,|\,B])$ then the system is inconsistent and has no solution.

(10) The homogenous system of linear equations $AX = 0$

(i) has the trivial solution, if $|A| \neq 0$.

(ii) has a non trivial solution, if $|A| = 0$.

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25. If $\omega = cis\dfrac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(1) 1 (2) 2 (3) 3 (4) 4

SUMMARY

In this chapter we studied

Rectangular form of a complex number is $x+iy$ (or $x+yi$), where x and y are real numbers.

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if $\text{Re}(z_1) = \text{Re}(z_2)$ and $\text{Im}(z_1) = \text{Im}(z_2)$. That is $x_1 = x_2$ and $y_1 = y_2$.

The conjugate of the complex number $x+iy$ is defined as the complex number $x-iy$.

Properties of complex conjugates

(1) $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

(2) $\overline{z_1-z_2} = \overline{z_1} - \overline{z_2}$

(3) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

(4) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$

(5) $\text{Re}(z) = \dfrac{z+\overline{z}}{2}$

(6) $\text{Im}(z) = \dfrac{z-\overline{z}}{2i}$

(7) $\overline{\left(z^n\right)} = \left(\overline{z}\right)^n$, where n is an integer

(8) z is real if and only if $z = \overline{z}$

(9) z is purely imaginary if and only if $z = -\overline{z}$

(10) $\overline{\overline{z}} = z$

If $z = x+iy$, then $\sqrt{x^2+y^2}$ is called modulus of z. It is denoted by $|z|$.

Properties of Modulus of a complex number

(1) $|z| = |\overline{z}|$

(2) $|z_1+z_2| \leq |z_1| + |z_2|$ (Triangle inequality)

(3) $|z_1 z_2| = |z_1||z_2|$

(4) $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$

(5) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, $z_2 \neq 0$

(6) $|z^n| = |z|^n$, where n is an integer

(7) $\text{Re}(z) \leq |z|$

(8) $\text{Im}(z) \leq |z|$

Formula for finding square root of a complex number

$$\sqrt{a+ib}=\pm\left(\sqrt{\frac{|z|+a}{2}}+i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right), \text{ where } z=a+ib \text{ and } b\neq 0.$$

Let r and θ be polar coordinates of the point $P(x,y)$ that corresponds to a non-zero complex number $z=x+iy$. The polar form or trigonometric form of a complex number P is

$$z=r(\cos\theta+i\sin\theta).$$

Properties of polar form

Property 1: If $z=r(\cos\theta+i\sin\theta)$, then $z^{-1}=\frac{1}{r}(\cos\theta-i\sin\theta)$.

Property 2: If $z_1=r_1(\cos\theta_1+i\sin\theta_1)$ and $z_2=r_2(\cos\theta_2+i\sin\theta_2)$,
then $z_1z_2=r_1r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$.

Property3: If $z_1=r_1(\cos\theta_1+i\sin\theta_1)$ and $z_2=r_2(\cos\theta_2+i\sin\theta_2)$,
then $\frac{z_1}{z_2}=\frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$.

de Moivre's Theorem

(a) Given any complex number $\cos\theta+i\sin\theta$ and any integer n,

$$(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$$

(b) If x is rational, then $\cos x\theta+i\sin x\theta$ in one of the values of $(\cos\theta+i\sin\theta)^x$

The n^{th} roots of complex number $z=r(\cos\theta+i\sin\theta)$ are

$$z^{1/n}=r^{1/n}\left(\cos\left(\frac{\theta+2k\pi}{n}\right)+i\sin\left(\frac{\theta+2k\pi}{n}\right)\right),\ k=0,1,2,3,\ldots,n-1.$$

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6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$

7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(1) 2 (2) 4 (3) 1 (4) ∞

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(1) $a \ge 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \le 0$

9. The polynomial $x^3 + 2x + 3$ has

(1) one negative and two imaginary zeros (2) one positive and two imaginary zeros

(3) three real zeros (4) no zeros

10. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r (-1)^r x^r$ is

(1) 0 (2) n (3) $< n$ (4) r

SUMMARY

In this chapter we studied

- Vieta's Formula for polynomial equations of degree 2,3, and n>3.
- **The Fundamental Theorem of Algebra** : A polynomial of degree $n \ge 1$ has at least one root in $\mathbb{C}$.
- **Complex Conjugate Root Theorem** : Imaginary (nonreal complex) roots occur as conjugate pairs, if the coefficients of the polynomial are real.
- **Rational Root Theorem** : Let $a_n x^n + \cdots + a_1 x + a_0$ with $a_n \ne 0$ and $a_0 \ne 0$, be a polynomial with integer coefficients. If $\frac{p}{q}$, with $(p, q) = 1$, is a root of the polynomial, then p is a factor of a_0 and q is a factor of a_n.
- Methods to solve some special types of polynomial equations like polynomials having only even powers, partly factored polynomials, polynomials with sum of the coefficients is zero, reciprocal equations.
- **Descartes Rule :** If p is the number of positive roots of a polynomial $P(x)$ and s is the number of sign changes in coefficients of $P(x)$, then $s - p$ is a nonnegative even integer.

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SUMMARY

Inverse Trigonometric Functions

Inverse sine function	Inverse cosine function	Inverse tangent function	Inverse cosecant function	Inverse secant function	Inverse cot function
Domain $[-1,1]$	Domain $[-1,1]$	Domain $\mathbb{R}$	Domain $(-\infty,-1]\cup[1,\infty)$	Domain $(-\infty,-1]\cup[1,\infty)$	Domain $\mathbb{R}$
Range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	Range $[0,\pi]$	Range $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	Range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$	Range $[0,\pi]-\left\{\frac{\pi}{2}\right\}$	Range $(0,\pi)$
not a periodic function	not a periodic function	not a periodic function	not a periodic function	not a periodic function	not a periodic function
odd function	neither even nor odd function	odd function	odd function	neither even nor odd function	neither even nor odd function
strictly increasing function	strictly decreasing function	strictly increasing function	strictly decreasing function with respect to its domain.	strictly decreasing function with respect to its domain.	strictly decreasing function
one to one function	one to one function	one to one function	one to one function	one to one function	one to one function

Properties of Inverse Trigonometric Functions

Property-I

(i) $\sin^{-1}(\sin\theta)=\theta$, if $\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta)=\theta$, if $\theta\in[0,\pi]$

(iii) $\tan^{-1}(\tan\theta)=\theta$, if $\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(iv) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta)=\theta$, if $\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\setminus\{0\}$

(v) $\sec^{-1}(\sec\theta)=\theta$, if $\theta\in[0,\pi]\setminus\left\{\frac{\pi}{2}\right\}$

(vi) $\cot^{-1}(\cot\theta)=\theta$, if $\theta\in(0,\pi)$

Property-II

(i) $\sin\left(\sin^{-1}x\right)=x$, if $x\in[-1,1]$

(ii) $\cos\left(\cos^{-1}x\right)=x$, if $x\in[-1,1]$

(iii) $\tan\left(\tan^{-1}x\right)=x$, if $x\in\mathbb{R}$

(iv) $\operatorname{cosec}\left(\operatorname{cosec}^{-1}x\right)=x$, if $x\in\mathbb{R}\setminus(-1,1)$

(v) $\sec\left(\sec^{-1}x\right)=x$, if $x\in\mathbb{R}\setminus(-1,1)$

(vi) $\cot\left(\cot^{-1}x\right)=x$, if $x\in\mathbb{R}$

Property-III (Reciprocal inverse identities)

(i) $\sin^{-1}\left(\frac{1}{x}\right)=\operatorname{cosec}x$, if $x\in\mathbb{R}\setminus(-1,1)$.

(ii) $\cos^{-1}\left(\frac{1}{x}\right)=\sec x$, if $x\in\mathbb{R}\setminus(-1,1)$

(iii) $\tan^{-1}\left(\frac{1}{x}\right)=\begin{cases}\cot^{-1}x & \text{if } x>0\\ -\pi+\cot^{-1}x & \text{if } x<0.\end{cases}$

Property-IV(Reflection identities)

(i) $\sin^{-1}(-x) = -\sin^{-1} x$, if $x \in [-1, 1]$.

(ii) $\tan^{-1}(-x) = -\tan^{-1} x$, if $x \in \mathbb{R}$.

(iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$, if $|x| \geq 1$ or $x \in \mathbb{R} \setminus (-1, 1)$.

(iv) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, if $x \in [-1, 1]$.

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x$, if $|x| \geq 1$ or $x \in \mathbb{R} \setminus (-1, 1)$.

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, if $x \in \mathbb{R}$.

Property-V (Cofunction inverse identities)

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ x \in [-1, 1]$. (ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ x \in \mathbb{R}$.

(iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2},\ x \in \mathbb{R} \setminus (-1, 1)$ or $|x| \geq 1$.

Property-VI

(i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$, where either $x^2 + y^2 \leq 1$ or $xy < 0$.

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$, where either $x^2 + y^2 \leq 1$ or $xy > 0$.

(iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$, if $x + y \geq 0$.

(iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right]$, if $x\,''\,y$.

(v) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$.

(vi) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$.

Property-VII

(i) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right),\ |x| < 1$ (ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\ x \geq 0$

(iii) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right),\ |x| \leq 1$

Property-VIII

(i) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1} x$, if $|x| \leq \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$.

(ii) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1} x$, if $\frac{1}{\sqrt{2}} \leq x \leq 1$.

Property-IX

(i) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$, if $0 \le x \le 1$. (ii) $\sin^{-1} x = -\cos^{-1}\sqrt{1-x^2}$, if $-1 \le x < 0$.

(ii) $\sin^{-1} x = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$, if $-1 < x < 1$. (iv) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$, if $0 \le x \le 1$.

(v) $\cos^{-1} x = \pi - \sin^{-1}\sqrt{1-x^2}$, if $-1 \le x < 0$. (vi) $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$, if $x > 0$.

Property-X

(i) $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$. (ii) $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\dfrac{1}{2}, 1\right]$.

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SUMMARY

(1) Equation of the circle in a standard form is $(x-h)^2+(y-k)^2=r^2$.

(i) Centre (h,k) (ii) radius 'r'

(2) Equation of a circle in general form is $x^2+y^2+2gx+2fy+c=0$.

(i) centre $(-g,-f)$ (ii) radius $=\sqrt{g^2+f^2-c}$

(3) The circle through the intersection of the line $lx+my+n=0$ and the circle

$x^2+y^2+2gx+2fy+c=0$ is $x^2+y^2+2gx+2fy+c+\lambda(lx+my+n)=0, \lambda\in\mathbb{R}^1$.

(4) Equation of a circle with (x_1,y_1) and (x_2,y_2) as extremities of one of the diameters is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

(5) Equation of tangent at (x_1,y_1) on circle $x^2+y^2+2gx+2fy+c=0$ is

$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

(6) Equation of normal at (x_1,y_1) on circle $x^2+y^2+2gx+2fy+c=0$ is

$yx_1-xy_1+g(y-y_1)-f(x-x_1)=0$.

Table 1

Tangent and normal

Curve	Equation	Equation of tangent	Equation of normal
Circle	$x^2+y^2=a^2$	(i) cartesian form $xx_1+yy_1=a^2$ (ii) parametric form $x\cos\theta+y\sin\theta=a$	(i) cartesian form $xy_1-yx_1=0$ (ii) parametric form $x\sin\theta-y\cos\theta=0$
Parabola	$y^2=4ax$	(i) $yy_1=2a(x+x_1)$ (ii) $yt=x+at^2$	(i) $xy_1+2y=2ay_1+x_1y_1$ (ii) $y+xt=at^3+2at$
Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	(i) $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ (ii) $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$	(i) $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2-b^2$ (ii) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$
Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	(i) $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$ (ii) $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$	(i) $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$ (ii) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

Table 2

Condition for the sine $y = mx + c$ to be a tangent to the Conics

Conic	Equation	Condition to be tangent	Point of contact	Equation of tangent
Circle	$x^2 + y^2 = a^2$	$c^2 = a^2(1+m^2)$	$\left(\dfrac{\mp am}{\sqrt{1+m^2}}, \dfrac{\pm a}{\sqrt{1+m^2}}\right)$	$y = mx \pm \sqrt{1+m^2}$
Parabola	$y^2 = 4ax$	$c = \dfrac{a}{m}$	$\left(\dfrac{a}{m^2}, \dfrac{2a}{m}\right)$	$y = mx + \dfrac{a}{m}$
Ellipse	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$c^2 = a^2m^2 + b^2$	$\left(\dfrac{-a^2m}{c}, \dfrac{b^2}{c}\right)$	$y = mx \pm \sqrt{a^2m^2 + b^2}$
Hyperbola	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$c^2 = a^2m^2 - b^2$	$\left(\dfrac{-a^2m}{c}, \dfrac{-b^2}{c}\right)$	$y = mx \pm \sqrt{a^2m^2 - b^2}$

Table 3

Parametric forms

Conic	Parametric equations	Parameter	Range of parameter	Any point on the conic
Circle	$x = a\cos\theta$ $y = a\sin\theta$	θ	$0 \le \theta \le 2\pi$	'θ' or $(a\cos\theta, a\sin\theta)$
Parabola	$x = at^2$ $y = 2at$	t	$-\infty < t < \infty$	't' or $(at^2, 2at)$
Ellipse	$x = a\cos\theta$ $y = b\sin\theta$	θ	$0 \le \theta \le 2\pi$	'θ' or $(a\cos\theta, b\sin\theta)$
Hyperbola	$x = a\sec\theta$ $y = b\tan\theta$	θ	$-\pi \le \theta \le \pi$ except $\theta = \pm\dfrac{\pi}{2}$	'θ' or $(a\sec\theta, b\tan\theta)$

Identifying the conic from the general equation of conic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

The graph of the second degree equation is one of a circle, parabola, an ellipse, a hyperbola, a point, an empty set, a single line or a pair of lines. When,

(1) $A = C = 1,\ B = 0, D = -2h,\ E = -2k,\ F = h^2 + k^2 - r^2$ the general equation reduces to $(x-h)^2 + (y-k)^2 = r^2$, which is a circle.

(2) $B=0$ and either A or $C=0$, the general equation yields a parabola under study, at this level.

(3) $A \neq C$ and A and C are of the same sign the general equation yields an ellipse.

(4) $A \neq C$ and A and C are of opposite signs the general equation yields a hyperbola

(5) $A=C$ and $B=D=E=F=0$, the general equation yields a point $x^2+y^2=0$.

(6) $A=C=F$ and $B=D=E=0$, the general equation yields an empty set $x^2+y^2+1=0$, as there is no real solution.

(7) $A \neq 0$ or $C \neq 0$ and others are zeros, the general equation yield coordinate axes.

(8) $A=-C$ and rests are zero, the general equation yields a pair of lines $x^2-y^2=0$.

ICT CORNER

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Open the Browser, type the URL Link given below (or) Scan the QR code. GeoGebra work book named "12th Standard Mathematics" will open. In the left side of the work book there are many chapters related to your text book. Click on the chapter named "Two Dimensional Analytical Geometry-II". You can see several work sheets related to the chapter. Select the work sheet "Conic Tracing"

SUMMARY

1. For a given set of three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, the scalar $(\vec{a}\times\vec{b})\cdot\vec{c}$ is called a **scalar triple product** of $\vec{a}, \vec{b}, \vec{c}$.
2. The **volume of the parallelepiped** formed by using the three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ as co-terminus edges is given by $\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right|$.
3. The scalar triple product of three non-zero vectors is zero if and only if the three vectors are **coplanar**.
4. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, if, and only if there exist scalars $r, s, t \in \mathbb{R}$ such that atleast one of them is non-zero and $r\vec{a} + s\vec{b} + t\vec{c} = \vec{0}$.
5. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{p}, \vec{q}, \vec{r}$ are any two systems of three vectors, and if $\vec{p} = x_1\vec{a} + y_1\vec{b} + z_1\vec{c}$, $\vec{q} = x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$, and, $\vec{r} = x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$, then $\left[\vec{p}, \vec{q}, \vec{r}\right] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}\left[\vec{a}, \vec{b}, \vec{c}\right]$.
6. For a given set of three vectors $\vec{a}, \vec{b}, \vec{c}$, the vector $\vec{a}\times(\vec{b}\times\vec{c})$ is called **vector triple product**.
7. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ we have $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$.
8. Parametric form of the vector equation of a straight line that passes through a given point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + t\vec{b}$, where $t \in \mathbb{R}$.
9. Cartesian equations of a straight line that passes through the point (x_1, y_1, z_1) and parallel to a vector with direction ratios b_1, b_2, b_3 are $\dfrac{x - x_1}{b_1} = \dfrac{y - y_1}{b_2} = \dfrac{z - z_1}{b_3}$.
10. Any point on the line $\dfrac{x - x_1}{b_1} = \dfrac{y - y_1}{b_2} = \dfrac{z - z_1}{b_3}$ is of the form $(x_1 + tb_1,\ y_1 + tb_2,\ z_1 + tb_3)$, $t \in \mathbb{R}$.
11. Parametric form of vector equation of a straight line that passes through two given points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + t\left(\vec{b} - \vec{a}\right)$, $t \in \mathbb{R}$.
12. Cartesian equations of a line that passes through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) are $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$.
13. If θ is the acute angle between two straight lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$, then $\theta = \cos^{-1}\left(\dfrac{\left|\vec{b}\cdot\vec{d}\right|}{\left|\vec{b}\right|\left|\vec{d}\right|}\right)$
14. Two lines are said to be coplanar if they lie in the same plane.
15. Two lines in space are called **skew lines** if they are not parallel and do not intersect
16. The shortest distance between the two skew lines is the length of the line segment perpendicular to both the skew lines.
17. The shortest distance between the two skew lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ is $\delta = \dfrac{\left|(\vec{c} - \vec{a})\cdot\left(\vec{b}\times\vec{d}\right)\right|}{\left|\vec{b}\times\vec{d}\right|}$, where $|\vec{b}\times\vec{d}| \neq 0$.

18. Two straight lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ intersect each other if $(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) = 0$

19. The shortest distance between the two parallel lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{b}$ is $d = \dfrac{|(\vec{c} - \vec{a}) \times \vec{b}|}{|\vec{b}|}$, where $|\vec{b}| \neq 0$

20. If two lines $\dfrac{x - x_1}{b_1} = \dfrac{y - y_1}{b_2} = \dfrac{z - z_1}{b_3}$ and $\dfrac{x - x_2}{d_1} = \dfrac{y - y_2}{d_2} = \dfrac{z - z_2}{d_3}$ intersect, then

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

21. A straight line which is perpendicular to a plane is called a normal to the plane.
22. The equation of the plane at a distance p from the origin and perpendicular to the unit normal vector $\hat{d}$ is $\vec{r} \cdot \hat{d} = p$ (normal form)
23. Cartesian equation of the plane in normal form is $lx + my + nz = p$
24. Vector form of the equation of a plane passing through a point with position vector $\vec{a}$ and perpendicular to $\vec{n}$ is $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$.
25. Cartesian equation of a plane normal to a vector with direction ratios a,b,c and passing through a given point (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.
26. Intercept form of the equation of the plane $\vec{r} \cdot \vec{n} = q$, having intercepts a, b, c on the x, y, z axes respectively is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$.
27. Parametric form of vector equation of the plane passing through three given non-collinear points is $\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t(\vec{c} - \vec{a})$
28. Cartesian equation of the plane passing through three non-collinear points is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

29. A straight will lie on a plane if every point on the line, lie in the plane and the normal to the plane is perpendicular to the line.
30. The two given non-parallel lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ are coplanar if $(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) = 0$.
31. Two lines $\dfrac{x - x_1}{b_1} = \dfrac{y - y_1}{b_2} = \dfrac{z - z_1}{b_3}$ and $\dfrac{x - x_2}{d_1} = \dfrac{y - y_2}{d_2} = \dfrac{z - z_2}{d_3}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

32. Non-parametric form of vector equation of the plane containing the two coplanar lines $\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{d}$ is $(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{d}) = 0$ or $(\vec{r} - \vec{c}) \cdot (\vec{b} \times \vec{d}) = 0$.
33. The acute angle θ between the two planes $\vec{r} \cdot \vec{n}_1 = p_1$ and $\vec{r} \cdot \vec{n}_2 = p_2$ is $\theta = \cos^{-1}\left(\dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$

34. If θ is the acute angle between the line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r} \cdot \vec{n} = p$, then $\theta = \sin^{-1}\left(\frac{\left|\vec{b} \cdot \vec{n}\right|}{\left|\vec{b}\right|\left|\vec{n}\right|}\right)$

35. The perpendicular distance from a point with position vector $\vec{u}$ to the plane $\vec{r} \cdot \vec{n} = p$ is given by $\delta = \frac{|\vec{u} \cdot \vec{n} - p|}{|\vec{n}|}$

36. The perpendicular distance from a point (x_2, y_1, z_1) to the plane $ax + by + cz = p$ is $\delta = \frac{|ax_1 + by_1 + cz_1 - p|}{\sqrt{a^2 + b^2 + c^2}}$.

37. The perpendicular distance from the origin to the plane $ax + by + cz + d = 0$ is given by

$$\delta = \frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$$

38. The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

39. The vector equation of a plane which passes through the line of intersection of the planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is given by $(\vec{r} \cdot \vec{n}_1 - d_1) + \lambda(\vec{r} \cdot \vec{n}_2 - d_2) = 0$, where $\lambda \in \mathbb{R}$ is an.

40. The equation of a plane passing through the line of intersection of the planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is given by

$$(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2z - d_2) = 0$$

41. The position vector of the point of intersection of the line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r} \cdot \vec{n} = p$ is $\vec{u} = \vec{a} + \left(\frac{p - (\vec{a} \cdot \vec{n})}{\vec{b} \cdot \vec{n}}\right)\vec{b}$, where $\vec{b} \cdot \vec{n} \neq \vec{0}$.

42. If $\vec{v}$ is the position vector of the image of $\vec{u}$ in the plane $\vec{r} \cdot \vec{n} = p$, then $\vec{v} = \vec{u} + \frac{2\left[p - (\vec{u} \cdot \vec{n})\right]}{|n|^2}\vec{n}$.

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